Complete the problems listed below in your blue book.
There are **no graphing calculators** allowed for this portion of the test.
To receive full credit, show all of your work.
When you are finished, fold your test and put it into your blue book.

1. *3 points* Write the x and y coordinate for each absolute maximum and absolute minimum of the function $f(x) = x^3 + 6x^2 - 15x$ on the closed interval $[-3, 6]$.

2. *16 points* A farmer has \$1500 available to build an E shaped fence along a straight river to enclose two identical rectangular pastures. She will not build a fence along the side of the pasture that borders the river. The material for the side of the fence running parallel to the river costs \$6 per foot, and the material for the three perpendicular sections costs \$5 per foot. Find the dimensions of the largest possible pasture.

   (a) Draw a picture of the enclosure, and label its dimensions with x and l, so that the length of the side running parallel to the river is l.

   (b) Write the objection equation.

   (c) Write the constraint equation.

   (d) Write the intervals of increasing and decreasing for the objective function. Write your answer in interval notation.

   (e) Write any relative minima or maxima for the objective function.

   (f) Write the intervals of concavity of the objective function (in interval notation). Write your answer in interval notation.

   (g) Sketch the graph of the objective function. Label any relative minima or maxima.

   (h) Determine the optimal values x and l.

3. *3 points* A large soup can is to be designed so that the can will hold 20π cubic inches of soup. Suppose that the radius of the base is x and the height of the can is h. Consider the problem of finding the values of x and h for which the amount of metal needed is as small as possible. You will **set up** this optimization problem.

   - The volume of the can is given by the formula: $\pi x^2 h$
   - The area of its circular top/bottom is πx^2
   - The circumference of the circular top/bottom is $2\pi x$

   (a) Draw a picture of the soup can and label its dimensions.

   (b) Write the objection equation.

   (c) Write the constraint equation.

4. *6 points* Suppose that $f(x)$ is a differentiable function satisfying the following properties:

   (i) $f(x) < 0$ (ii) $f'(x) = f(x)$ (iii) $f(0) = -1$

   (a) Write the intervals of increasing and decreasing for $f(x)$ (in interval notation).
   (b) Write the intervals of concavity for $f(x)$ (in interval notation).
   (c) Sketch the graph of $f(x)$, and label the point $f(0) = -1$.

5. *5 points* Consider the following differential equation: $f'(x) = x^2 - 4x + 3$

   (a) Draw the slope-field for the differential equation, for $0 \leqslant x \leqslant 4$.
   (b) Use your answer to the previous part to estimate the x-values at which the function $f(x)$ has a local minimum or local maximum.

6. *8 points* Approximately 100 bacteria are placed in a culture. Let $P(t)$ be the number of bacteria present in the culture after t hours, and suppose that $P(t)$ satisfies the differential equation: $P'(t) = .01P(t)$.

   (a) Find the solution to the differential equation above.
   (b) Suppose that after 2 hours, there are approximate 102 bacteria in the culture. What is the growth rate of the bacteria after 2 hours?
   (c) Write the equation of the tangent line to $P(t)$ for $t = 2$, and use it to estimate the number of bacteria in the culture after 3 hours.
   (d) Is your estimate from the previous part an over-estimate or an under-estimate? Explain your answer by sketching the graph of $P(t)$.

7. *4 points* Suppose that eighteen grams of a radioactive material disintegrates to six grams after one year. Write $M(t)$ for the mass of the material with respect to time (measured in years), and write M_0 for the initial mass of material.

   (a) Compute the decay constant for $M(t)$. (You do not need an exact number for your answer.)
   (b) What is the half-life of this material? (You do not need an exact number for your answer.)

8. *Extra Credit 5 points* Suppose that $g(x)$ is a differentiable function with $g(x) > 0$ for each x.

   (a) Compute the derivative of the function $f(x) = \ln(g(x))$. In your answer write $g'(x)$ for the derivative of $g(x)$.
   (b) Use logarithmic differentiation to compute the derivative of the function $g(x) = e^x(x-4)^8$.