Complete the problems listed below in your blue book.
There are **no graphing calculators** allowed for this portion of the test.
To receive full credit, show all of your work.
When you are finished, fold your test and put it into your blue book.

1. *5 points* Sketch the graph of the following function:

$$f(x) = \frac{x^2 - 8x + 12}{x - 6}.$$

Then, use your graph to compute the limit $\lim_{x\to 6} f(x)$.

2. *4 points* In each item below, sketch the graph of a possible function that fits the description listed. (You should have two graph-sketches, one for each item.)

   (a) The limit $\lim_{x\to 7} f(x)$ does not exist.

   (b) $f(x)$ is continuous at the point $x = 4$, and not differentiable at $x = 4$.

3. *6 points* Suppose that a ball is thrown straight into the air. Its vertical position (in meters) t seconds after the ball was thrown is given by the function $s(t) = -16t^2 + 64t + 1$. Regard "up" as the positive direction.

   (a) What is the average velocity of the ball over the time interval from $t = 1$ to $t = 3$?

   (b) What is the (instantaneous) velocity of the ball at time $t = 1$.

   (c) Suppose that after $t = 1$, the ball continued to travel with the velocity that it had at that instant. Use the equation of the tangent line (at $t = 1$) to give the ball's position after 5 seconds.

4. *4 points* Sketch the graph of a function near the point $x = 4$ so that it fits the description given below: $f(4) = 2$; $f'(4) = -1$; $x = 4$ is an inflection point with $f''(4) < 0$ when $x < 4$.

5. *15 points* Consider the function $f(x) = 3x^4 - 4x^3$.

   (a) Find the intervals of increasing and decreasing for $f(x)$. Write your final answer in interval notation. For example, if $f(x)$ is increasing on the interval $2 < x < 6$, then write"Increasing: $(2, 6)$".

   (b) Find any relative maxima or minima for $f(x)$. Include both the x and y coordinates.

   (c) Find the intervals of concavity for $f(x)$. Write your final answer in interval notation.

   (d) Find any inflection points for $f(x)$. Include both the x and y coordinates.

   (e) Sketch the graph of $f(x)$, indicating all relative minima and maxima and inflection points.

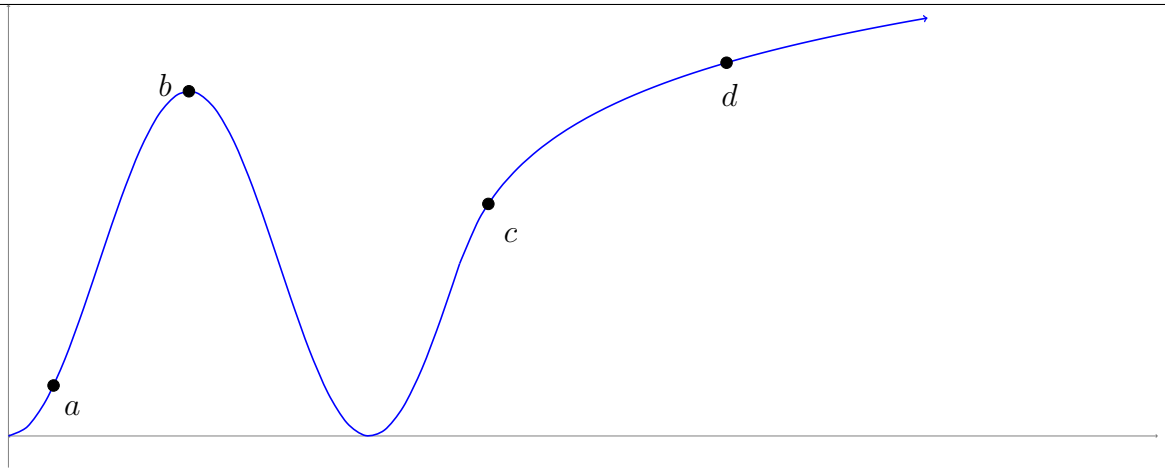

Figure 1: The above graph depicts the position of a car traveling in a straight line.

6. *6 points* Refer to Figure 1, which shows the position of a car moving in a straight line. In the figure, the x-axis denotes time, and the y-axis denotes the position of the car relative to a fixed finish-line (at $y = 0$). Assume that the positive direction is the direction that takes the car away from the finish-line.

   (a) At which point(s) is the car traveling fastest (in either the positive or negative direction): a, b, c or d? Explain your answer using the slope of the tangent line.

   (b) Draw in tangent lines near the point c, and use them describe how the velocity is changing near c. Specifically, is the velocity increasing or decreasing near c?

   (c) At the point d, describe the acceleration of the car. Specifically, does the car have positive acceleration or negative acceleration?

7. *Extra Credit* Consider a smooth curve $y = f(x)$ with no undefined points. If $f(x)$ has two relative maximum points, must it have a relative minimum point? Explain your answer in words or with a picture.