Complete the problems listed below in your blue book. To receive full credit, show all of your work. In graph sketches, label at least on point on the graph. When you are finished, fold your test and put it into your blue book.

- 1. 5 points Suppose that the population of a certain country is currently 7 million. The growth of this population attributable to an excess of births over deaths is 10% per year. Further, the country is experiencing emigration at the rate of 15,000 people per year.
 - (a) Write the difference equation for this problem *and* the solution to the difference equation.
 - (b) Assume that there are no excess births over deaths. Write the difference equation and the solution to the difference equation.
- 2. 5 points For each of the following, sketch the graph of a difference equation that fits the description listed.
 - (a) Oscillating and unbounded (b) Monotonic and |a| < 1
- 3. 12 points Allen makes an initial deposit of y_0 dollars into a savings account paying an annual interest rate of 12%, compounded monthly. He plans to withdraw \$650 at the end of each month.
 - (a) Write the difference equation for this problem.
 - (b) Suppose that Allen makes an initial deposit of \$50,000. Sketch the graph of the solution to the difference equation. Label the point y_0 and the line $y = \frac{b}{1-a}$ on your graph.
 - (c) Suppose that Allen makes an initial deposit of \$500,000. Sketch the graph of the solution to the difference equation. Label the point y_0 and the line $y = \frac{b}{1-a}$ on your graph.
 - (d) What is the smallest value for y_0 such that Allen's account will never run out of money? Round your answer to the nearest dollar. Sketch the graph of the solution to the difference equation for this value of y_0 .
- 4. 8 points Doreen borrows 90,000 from a bank that charges interest at an annual rate of 10 percent, compounded monthly. She plans to make a monthly payment of b dollars.
 - (a) Write the difference equation for this problem.
 - (b) Calculate the monthly payment that she would have to make in order to pay off the loan after exactly 30 years. (Round your answer to the nearest cent.)

- 5. 13 points For each of the following functions y = f(x) listed below:
 - (i) Compute the derivative of f(x) (as a function).
 - (ii) Write the equation of the tangent line to f(x) at the point indicated.
 - (iii) Choose *one* function, and sketch the graph of f(x) and the tangent line at the point indicated.

(a)
$$f(x) = 1/x$$
, $x = 3$ (b) $f(x) = \sqrt[7]{(x^3)}$, $x = 1$ (c) $f(x) = x^2$, $x = -2$.



Figure 1: Use the above graph in Problem 6.

- 6. 3 points Use Figure 1 to write the equation of the tangent line at x = 1 for f(x).
- 7. 4 points The definition of the derivative of the function f(x) at x_0 as a limit is:

$$\lim_{h \to 0} \frac{f(x_0 + h) - f(x_0)}{h}$$

Use the limit above to compute the derivative of $f(x) = x^2 - 4x + 3$ at the point $x_0 = 2$.

8. Extra credit Determine whether the function $f(x) = x^2 - 4x + 3$ is increasing, decreasing or constant near the point $x_0 = 2$. Justify your answer.