Test 1 Review Sheet

1.

(a) $F(x) = -\frac{1}{3}e^{-3x} + C$

(b) $F(x) = \frac{4}{2}x^2 + C$

$= 2x^2 + C$

(c) $F(x) = \frac{x^{10}}{10} + C$

(d) $F(x) = \frac{3}{6}x^6 + x + C$

$= \frac{1}{2}x^6 + x + C$

(e) Typo: $f(x) = \sqrt{e^x}$

$= (e^x)^{1/2} = e^{\frac{1}{2}\cdot x}$

$F(x) = 2 \cdot e^{(1/2)x} + C$

(f) $f(x) = 7 \cdot \frac{1}{x}$

$F(x) = 7 \cdot \ln|x| + C$

(g) $f(x) = 8 \cdot x^{1/3}$

$F(x) = 8 \cdot \frac{3}{4}x^{4/3} + C$

$= 6 \cdot x^{4/3} + C$

(h) ~~Typo, Sorry!~~ $f(x) = \frac{7}{e^{2x}}$

$\cancel{f(x) = e^{(1/2)x}}$ $= 7 \cdot e^{-2x}$

$F(x) = \cancel{\frac{7}{1/2} e^{(1/2)x} + C}$

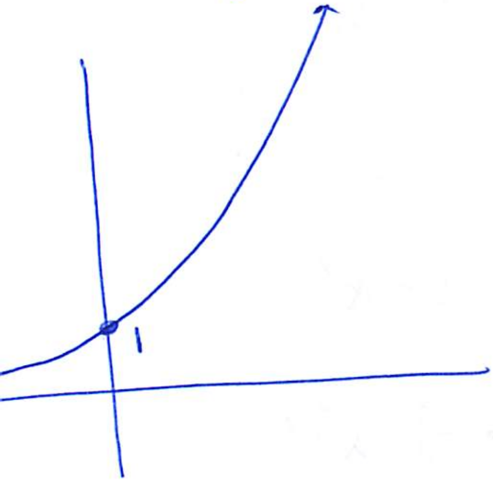
$\frac{7}{-2}e^{-2x} + C$

(i) $f(x) = 2 \cdot x^{-1/2} + 2 \cdot x^{1/2}$

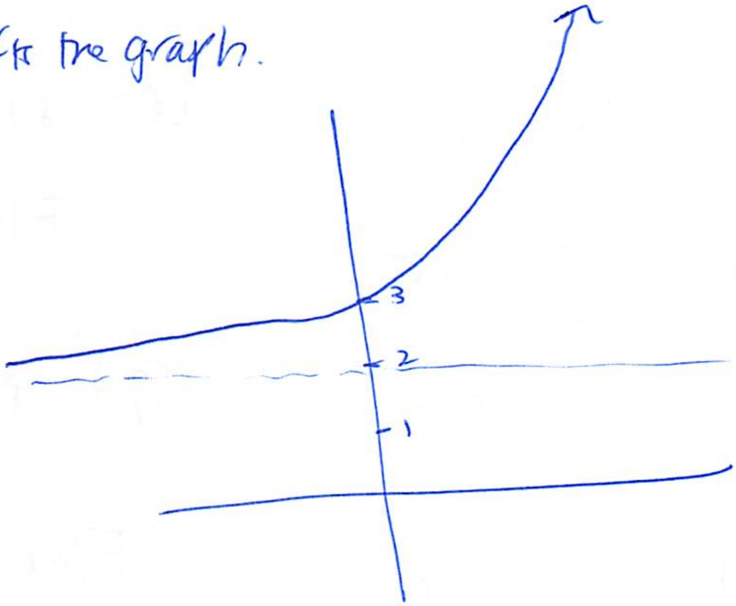
$F(x) = 2 \cdot 2x^{1/2} + 2 \cdot \frac{2}{3}x^{3/2} + C$

$= 4x^{1/2} + \frac{4}{3}x^{3/2} + C$

Each antiderivative has the form $F(x) = e^x + C$ and changing C vertically shifts the graph.

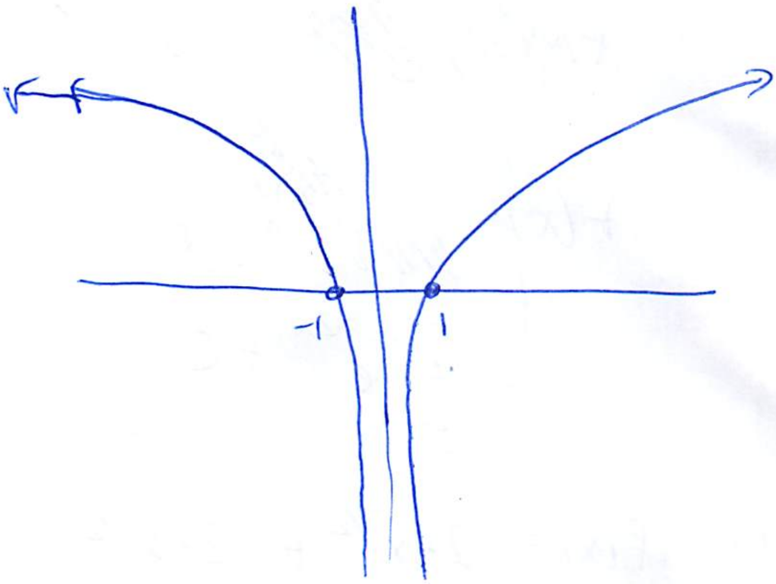


$F(x) = e^x$



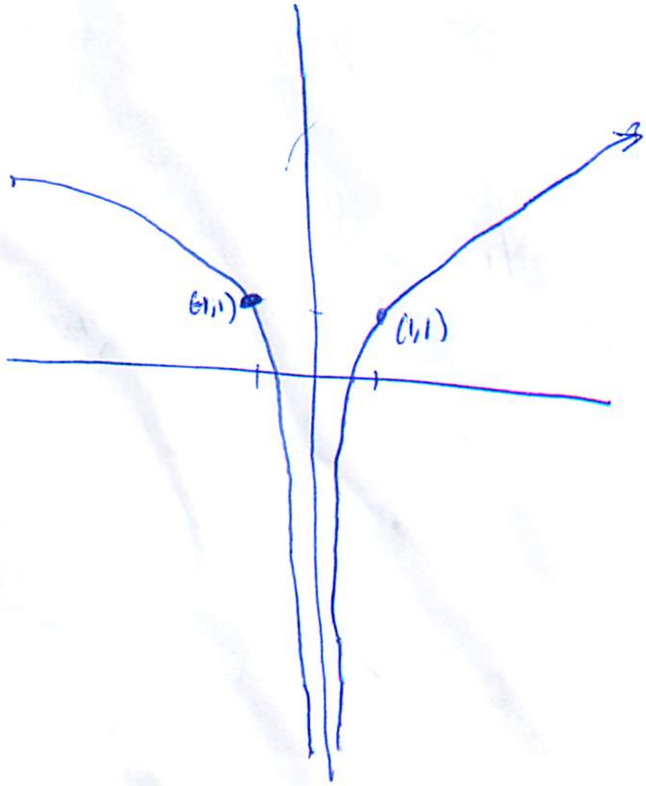
$F(x) = e^x + 2$

3. Each antiderivative has the form $F(x) = \ln|x| + C$



$F(x) = \ln|x| + 0$

(sorry, In this graph there should be symmetry about the y-axis)



$F(x) = \ln|x| + 1$

5(b) $f(x) = 2x+1$ that's a two, sorry that its a little messy!

$F(x) = x^2 + x + C$

$F(0)=5 \longrightarrow$ $5 = 0^2 + 0 + C$

$5 = C$

$$\boxed{F(x) = x^2 + x + 5}$$

5(c) $F(x) = \frac{1}{7}e^{7x} + C$

$F(0) = 10 \rightarrow$ $\frac{1}{7} \cdot e^{7 \cdot 0} + C = 10$

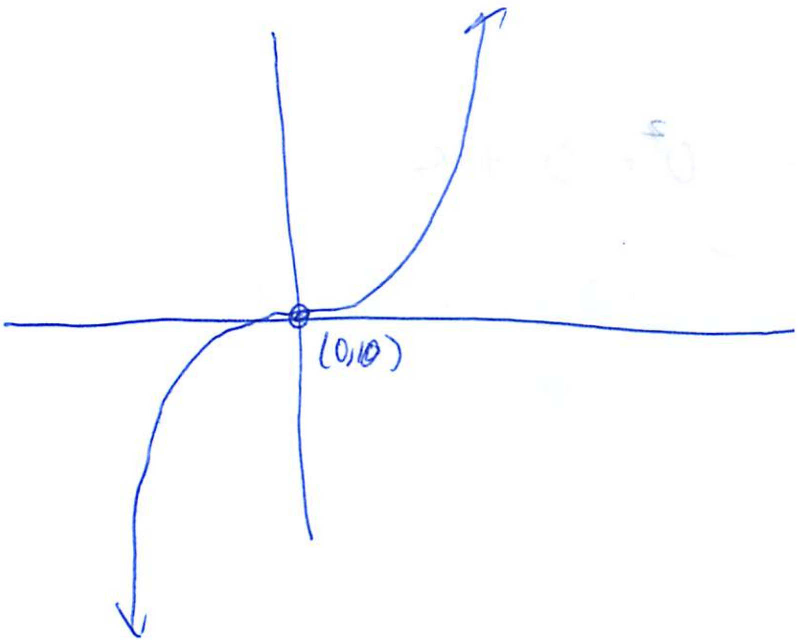
$\frac{1}{7}e^0 + C = 10$

$\frac{1}{7} + C = 10$

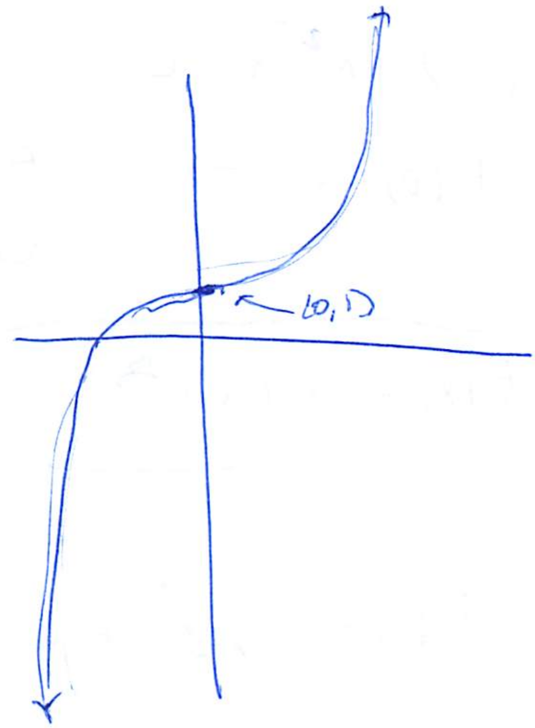
$C = \frac{70}{7} - \frac{1}{7} = \frac{69}{7}$

$$\boxed{F(x) = \frac{1}{7}e^{7x} + \frac{69}{7}}$$

4. All antiderivatives have the form $F(x) = x^3 + C$



$F(x) = x^3$



$F(x) = x^3 + 1$

4.5

(a) $f(x) = x^2 + x^{1/2}$

$F(x) = \frac{1}{3}x^3 + \frac{2}{3}x^{3/2} + C$

$F(1) = 3 \rightarrow \quad 3 = \frac{1}{3}\cdot 1^3 + \frac{2}{3}1^{3/2} + C$

$3 = 1/3 + 2/3 + C$

$3 = 1 + C$

$\underline{2 = C}$

$$\boxed{F(x) = \frac{1}{3}x^3 + \frac{2}{3}x^{3/2} + 2}$$

6.

(a) $\int 7\left(\frac{x^2}{3} + 3x\right) dx$

$= 7 \int \frac{x^2}{3} + 3x\, dx$

$= 7\left(\int \frac{x^2}{3} dx + \int 3x\, dx\right)$

$= 7\left(\frac{x^3}{3\cdot 3} + \frac{3x^2}{2}\right) + C$

$= \boxed{\frac{7x^3}{9} + \frac{21}{2}x^2 + C}$

(b) $\int \frac{1}{e^x} dx = \int e^{-x} dx$

$= \boxed{-e^{-x} + C}$

(c) $\int \frac{x^3}{3} + \frac{3}{x^3} + \frac{3}{x}\, dx = \int \frac{x^3}{3} dx + \int \frac{3}{x^3} dx + \int \frac{3}{x} dx$

$= \int \frac{x^3}{3} dx + \int 3 \cdot x^{-3} dx + 3\int \frac{1}{x} dx$

$= \boxed{\frac{x^4}{4\cdot 3} + \frac{3x^{-2}}{-2} + 3\ln|x| + C}$

6(d)

$$\int \frac{3x - 2x^3 + 4x^5}{4x^7}\,dx$$

$$= \int \frac{3x}{4x^7} + \frac{-2x^3}{4x^7} + \frac{4x^5}{4x^7}\,dx$$

$$= \frac{3}{4}\int x^{1-7}\,dx + \frac{-2}{4}\int x^{3-7}\,dx + \int x^{5-x^7}\,dx$$

$$= \frac{3}{4}\int x^{-6}\,dx + -\tfrac{1}{2}\int x^{-4}\,dx + \int x^{-2}\,dx$$

$$= \frac{3}{4}\cdot\frac{-1}{5}x^{-5} + \frac{-1}{2}\cdot\frac{-1}{3}x^{-3} + -1x^{-1} + C$$

two neg.'s cancel

$$= \frac{-3}{20}x^{-5} + \frac{+1}{6}x^{-3} + -x^{-1} + C$$

$$= \boxed{\frac{-3}{20}x^{-5} + \frac{1}{6}x^{3} - x^{-1} + C}$$

7. $u = 2x - 1$

(a) $\dfrac{du}{dx} = 2 \to du = 2\,dx$

$$\int (2x-1)^7\, 2\,dx = \int u^7\,du$$

$$= \frac{1}{8}u^8 + C$$

$$= \boxed{\frac{1}{8}(2x-1)^8 + C}$$

(b) $u = x^2 + 2x + 3$, $\dfrac{du}{dx} = 2x + 2$, $du = (2x+2)\,dx$

multiply

$$\int (x^2+2x+3)^8 (x+1)\,dx = \frac{1}{2}\int 2(x^2+2x+3)^8(x+1)\,dx$$

$$= \frac{1}{2}\int (x^2+2x+3)^8 (2x+2)\,dx$$

$$= \frac{1}{2}\int u^8\,du = \frac{1}{2}\cdot\frac{1}{9}u^9 + C$$

$$= \boxed{\frac{1}{18}(x^2+2x+3)^9 + C}$$

(c) $\int \frac{\sqrt{\ln(x)}}{x}\,dx$

$= \int \frac{(\ln(x))^{1/2}}{x}\,dx \qquad u = \ln(x)$

$\frac{du}{dx} = \frac{1}{x}, \quad du = \frac{dx}{x}$

$= \int (\ln(x))^{1/2} \cdot \frac{dx}{x} = \int u^{1/2}\,du$

$= \frac{2}{3}u^{3/2} + C$

$= \boxed{\frac{2}{3}(\ln(x))^{3/2} + C}$

(d) $\int \frac{x^4}{(x^5+11)}\,dx \qquad u = x^5+11$

$\frac{du}{dx} = 5x^4, \quad du = 5x^4\,dx$

$\int \frac{x^4}{(x^5+11)}\,dx = \frac{1}{5}\int \frac{5x^4}{(x^5+11)}\,dx = \frac{1}{5}\int \frac{du}{u} = \frac{1}{5}\int \frac{1}{u}\cdot du$

$= \frac{1}{5}\ln|u| + C = \boxed{\frac{1}{5}\ln|x^5+11| + C}$

(6e) $\int e^{-x^2} 2x\,dx$

$u = -x^2$

$\frac{du}{dx} = -2x \rightarrow du = -2x\,dx$

$$\int e^{-x^2} 2x\,dx = -\int -e^{-x^2} 2x\,dx$$

$$= -\int e^{-x^2} \cdot (-2x)\,dx$$

$$= -\int e^{u}\,du$$

$$= -e^{u} + C$$

$$= \boxed{-e^{-x^2} + C}$$

(6f) $\int \left(\frac{3x^2+1}{\sqrt[5]{x^3+x+10}} \right) dx = \int (x^3+x+10)^{-1/5} \cdot (3x^2+1)\,dx$

$u = x^3+x+10$

$\frac{du}{dx} = 3x^2+1 \rightarrow du = (3x^2+1) \cdot dx$

$$= \int u^{-1/5}\,du$$

$$= \frac{5}{4} u^{4/5} + C = \boxed{\frac{5}{4} \cdot (x^3+x+10)^{4/5} + C}$$

8.

(a) $\int_{-2}^{2} x^5 + x^3 + x \, dy$

$= \frac{1}{6}x^6 + \frac{1}{4}x^4 + \frac{1}{2}x^2 \Big|_{-2}^{2}$

$= \left(\frac{1}{6}(2)^6 + \frac{1}{4}(2^4) + \frac{1}{2}(2)^2\right) - \left(\frac{1}{6} \cdot (-2)^6 + \frac{1}{4}(-2)^4 + \frac{1}{2}(-2)^2\right)$

$= \boxed{0}$

(b) $\int_{-2}^{2} x^2 + 1 \, dx = \frac{1}{3}x^3 + x \Big|_{-2}^{2}$

$\frac{1}{3}2^3 + 2 - \left(\frac{1}{3}(-2)^3 + (-2)\right)$

$= \boxed{\frac{28}{3}}$

9. (a)

$$\int_0^3 -32t + 74 \, dt$$

$$= \frac{-32t^2}{2} + 74t \Big|_0^3$$

$$= -16t^2 + 74t \Big|_0^3$$

$$= -16 \cdot 3^2 + 74 \cdot 3 - \left(-16 \cdot 0^2 + 74 \cdot 0\right)$$

$$= \underline{\underline{78}}$$

(b) $\int -32t + 74 = s(t)$

$-16t^2 + 74t + C = s(t)$

Also $s(0) = 6 \longrightarrow$ $-16 \cdot 0^2 + 74 \cdot 0 + C = 6$

$C = 6$

$s(t) = -16t^2 + 74t + 6$

$s(3) = -16 \cdot 9 + 74 \cdot 3 + 6$

$= \boxed{84}$

#10 We have $R(t) = M'(t)$ and we want $M(10) - M(0)$.

So, we set up a definite integral!

$$\int_0^{10} -e^{-.1t}\,dt = -\int_0^{10} e^{-.1t}\,dt$$

$$= -1 \cdot \frac{1}{-.1} \cdot e^{-.1t} \Big|_0^{10}$$

neg's cancel and $\frac{1}{.1} = 10$

$$= 10e^{-.1t} \Big|_0^{10}$$

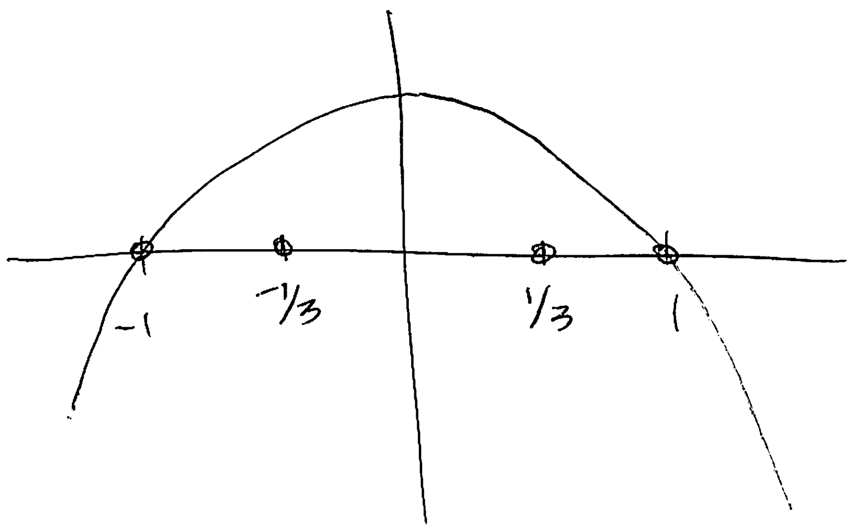
$$10 \cdot e^{-.1 \cdot 10t} - 10 \cdot e^{-.1 \cdot 0}$$

$$= \boxed{10e^{-1} - 10}$$

11. $f(x) = 1 - x^2$

Δx or width $= \dfrac{\text{total length}}{3}$

$= \dfrac{1-(-1)}{3} = 2/3$



(a) Right endpts:

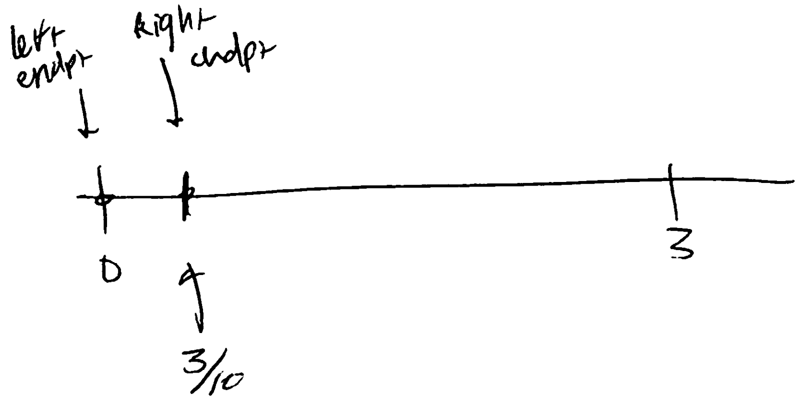
$(2/3) \cdot f(-1/3) + (2/3)\, f(1/3) + 2/3 f(1)$

$= 2/3 \cdot (1 - (-1/3)^2) + 2/2 \cdot (1 - (1/3)^2) + 2/3 \cdot (1 - (-1)^2)$

(b) Left endpts:

$2/3 \cdot (1 - (-1)^2) + 2/3 (1 - (-1/3)^2) + 2/3 (1 - (1/3)^2)$

12. $\dfrac{\text{total length}}{\#\text{ of rect}} = \dfrac{3-0}{10} = 3/10$



Right endpts : this 1st rectangle has area: $(3/10) \cdot f(3/10)$

$= 3/10 \cdot (3/10)^3$

left endpts : this 1st rectangle has area: $3/10 \cdot f(0)$

$= 3/10 \cdot 0$

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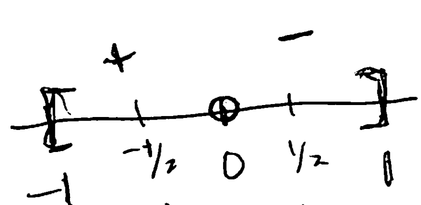
13. (a) Need to find where our function is positive and where it is negative.

$f(x)=0$, solve & test pts on either side.

$x(x^2-1)=0 \longrightarrow x = 0, -1, 1$



test at $x = -1/2$
$x = 1/2$

1 & −1 are endpts for the integral, so you dont need to test around these.

at $-1/2 \to f(-1/2) = -1/2(-3/4)$ which is positive

$f(1/2) = 1/2(-3/4)$ which is neg. on $[-1,1]$

Set up integral:

$$\int_{-1}^{0} x(x^2-1)\,dx + -\int_{0}^{1} x(x^2-1)\,dx$$

multiply by neg 1 b/c our function is negative but we want positive area

$$= \int_{-1}^{0} x^3 - x\,dx + \int_{0}^{1} x - x^3\,dx$$

$$= \frac{x^4}{4} - \frac{x^2}{2}\Big|_{-1}^{0} + \frac{x^2}{2} - \frac{x^4}{4}\Big|_{0}^{1}$$

$$= 0 - \left(\frac{1}{4} - \frac{1}{2}\right) + = 1/2 - 1/4$$

$$= -\left(-1/4\right) + = 1/4$$

$$= 1/4 + 1/4 = \boxed{1/2}$$

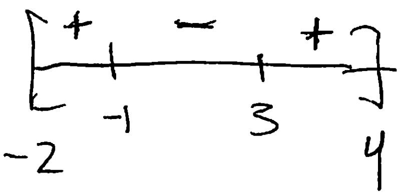
13 b

Find where $f(x) = x^2 - 2x - 3$ is positive & where its neg. on the interval $[-2, 4]$

$0 = x^2 - 2x - 3$

$0 = (x-3)(x+1)$

$x = -1, 3$



test points on either side.

I tested -1.5, 0, & 3.5

Set up:

$$\int_{-2}^{-1} x^2 - 2x \cancel{8} - 3\,dx + -\int_{-1}^{3} x^2 - 2x - 3\,dx + \int_{3}^{4} x^2 - 2x - 3\,dx$$

Answer: $7/3 + \frac{32}{3} + 7/3$

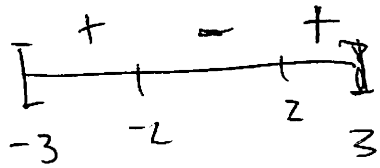
$= \boxed{\frac{46}{3}}$

13. Need to find when $f(x) = x^2 - 4$ is positive & neg on the interval $[-3, 3]$

$0 = x^2 - 4$

$0 = (x+2)(x-2)$

$x = 2, -2$



I tested $-2.5, 0, 2.5$

Set up:

$$\int_{-3}^{-2} x^2 - 4\,dx + -\int_{-2}^{2} x^2 - 4\,dx + \int_{2}^{3} x^2 - 4\,dx$$

$$7/3 + \frac{32}{3} + 7/3$$

$$= \boxed{\frac{46}{3}}$$