Test 3 Review solutions

1. $f(x) = 2x^3 - 3x^2 - 12x + 1 \quad [-2, 3]$

$$0 = 6x^2 - 6x - 12$$

$f'(x) = 0 \longrightarrow$ $0 = 6(x^2 - x - 2)$

$0 = 6(x-2)(x+1)$ divide out 6

$$0 = (x-2)(x+2)$$

$$x = 2, -1$$

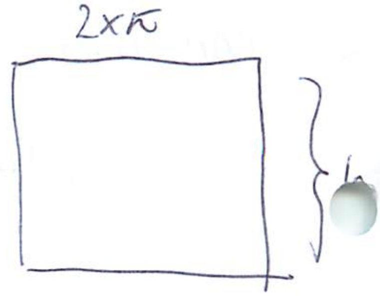
Test $x = 2, -1$ and endpts: $x = -2, 3$ in original function

x	$f(x)$
-2	-3
-1	8
2	-19
3	-8

- Abs max at $x = -1$ $y = 8$

(Abs min at $x = 2$. $y = -19$)

4. (a) objective function



Surface Area:

$SA = (2\pi x) \cdot h + 2\pi x^2$

(b) constraint: $(h)(\pi x^2) = 16\pi$

$$h = \frac{16\cancel{\pi}}{\cancel{\pi} x^2} = \frac{16}{x^2}$$

(c) $$SA(x) = \frac{2\pi x \cdot 16}{x^2} + 2\pi x^2$$

$$= \frac{32\pi}{x} + 2\pi x^2$$

$$SA'(x) = \frac{-32\pi}{x^2} + 4\pi x$$

$$0 = \frac{-32\pi}{x^2} + 4\pi x$$

$$\frac{32\pi}{x^2} = 4\pi x$$

$$\frac{32\pi}{4\pi} = \frac{4\pi x^3}{4\pi}$$

$8 = x^3$

$2 = x$

$$\begin{array}{ccccc} & - & & + & \\ 0 & & 2 & & +\infty \end{array}$$

Decreasing: $(0, 2)$ Increasing: $(2, +\infty)$

4 (c): relative min at $x = 2$

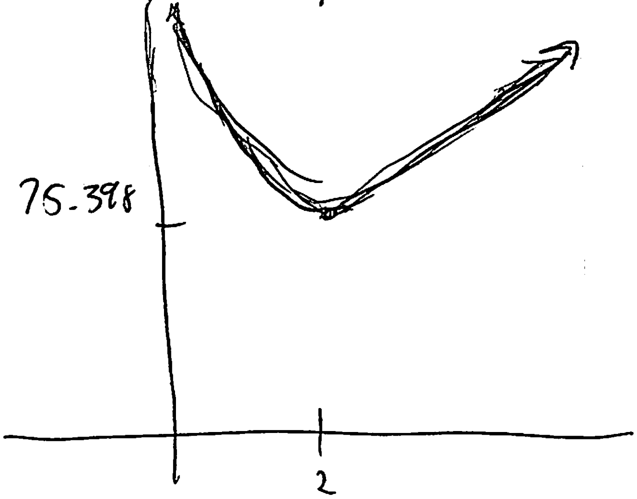
$SA(2) = \approx 75.398$

Concavity: $SA''(x) = \frac{64\pi}{x^3} + 4\pi$

When $x > 0 \rightarrow SA'' > 0$

Therefore, SA is concave up $(0, +\infty)$

(d)



(e) Abs min $x = 2$

$h = 4$

$SA = 75.398$

3. (a) Increasing / Decreasing : want to know when $f'(x)$ is positive / negative

$f'(x) = -\underbrace{f(x)}_{\text{always positive}}$

positive times negative $\rightarrow$ $f'(x)$ is negative!

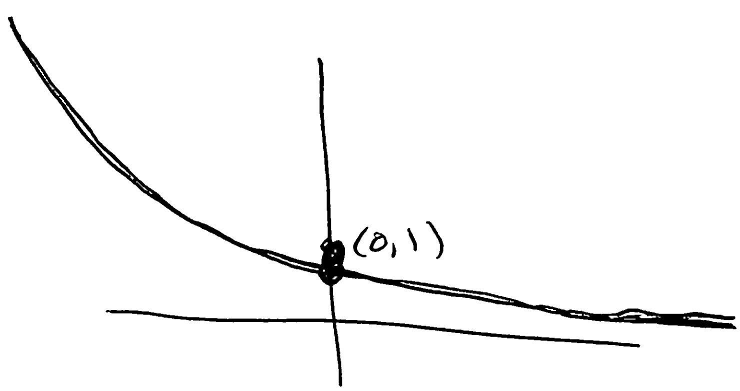
Decreasing: $(-\infty, +\infty)$
Increasing : Never

(b) Concavity : Need to find where $f''(x)$ is positive/neg.

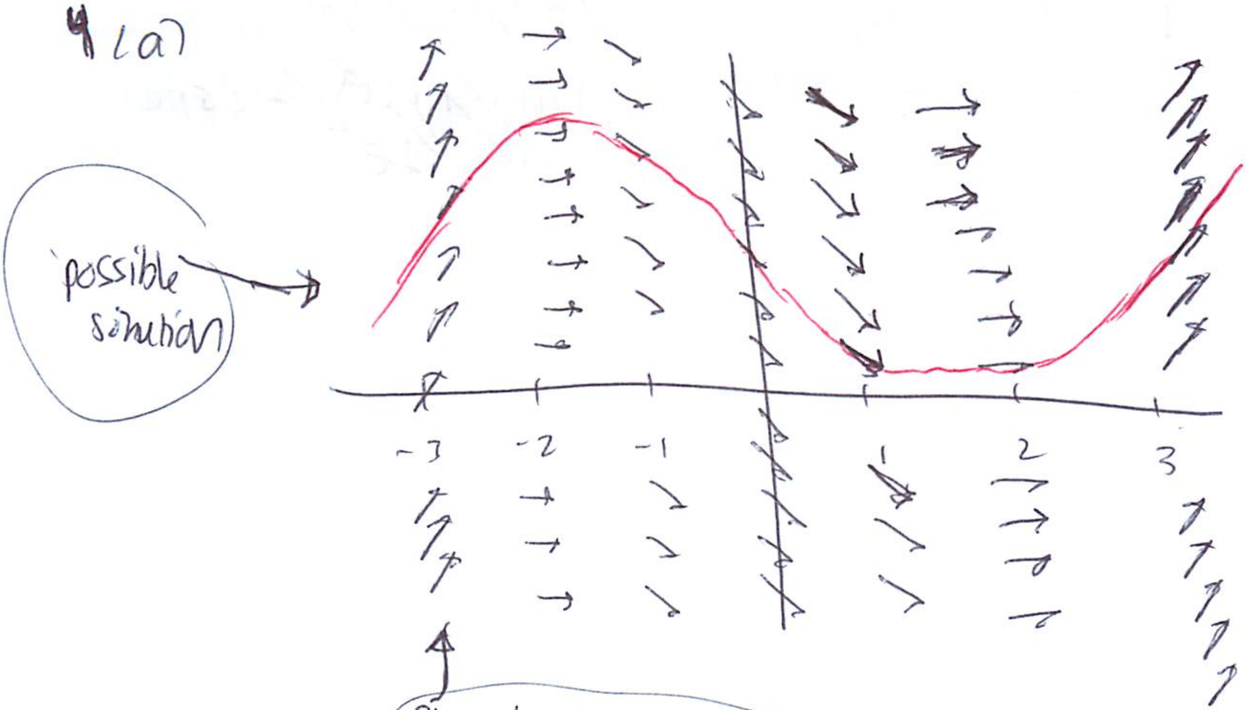
$f''(x) = (f'(x))' = (-f(x))' = -f'(x)$
$= -(-f(x))$
$= \underline{f(x)}$ always positive

concave up: $(-\infty, +\infty)$
concave down: Never

(c)



4(a)



Slope here is $f'(-3) = 5$ very steep + positive

4(b)

relative max at $x = -2$
relative min at $x = 2$

5. (a) $P(t) = 100 e^{0.1t}$

Remember that solutions to the differential eqn $P'(t) = k P(t)$ look like

$P(t) = P_0 e^{kt}$

(b) $P(3) = 100 \cdot e^{(0.1)\cdot 3}$

≈ 134.98

(c) Growth Rate = Derivative!

$P'(t) = .1 P(t)$ (Given!)

plug in 3:

$P'(3) = (.1) P(3)$

$= .1 \cdot (134.98)$

$\approx \underline{13.498}$

(d) Eqn of tangent line:

$y - 134.98 = (13.498)(x-3)$

plug in 4: $y = (13.498)(4-1) + 134.98 = 148.487$

5(e) This is an underestimate.