1. Find the absolute maximum of the function $f(x) = 2x^3 - 3x^2 - 12x + 1$ on the closed interval $[-2, 3]$.

2. A large soup can is to be designed so that the can will hold 16π cubic inches of soup. Suppose that the radius of the base is x and the heigh of the can is h. Find the values of x and h for which the amount of metal needed is as small as possible.

   (a) Draw a picture of the soup can and label its dimensions.

   (b) Write the objection equation.

   (c) Write the constraint equation.

   (d) Find the intervals of increasing and decreasing, relative minima or maxima, and the intervals of concavity.

   (e) Sketch the graph of the objective function.

   (f) Determine the optimal values for the dimensions.

3. Suppose that $f(x)$ is a differentiable function satisfying the following properties:

   (a) (i) $f(x) > 0$ (b) (ii) $f'(x) = -f(x)$ (c) (iii) $f(0) = 1$

   (a) Write the intervals of increasing and decreasing for $f(x)$ (in interval notation).

   (b) Write the intervals of concavity for $f(x)$ (in interval notation).

   (c) Sketch the graph of $f(x)$, and label the point $f(0) = 1$.

4. Consider the following differential equation: $f'(x) = x^2 - 4$

   (a) Draw the slope-field for the differential equation, for $-3 < x < 3$.

   (b) Use your answer to the previous part to list the x-values at which the function $f(x)$ has a local minimum or local maximum.

5. Approximately 100 bacteria are placed in a culture. Let $P(t)$ be the number of bacteria present in the culture after t hours, and suppose that $P(t)$ satisfies the differential equation: $P'(t) = .1P(t)$.

   (a) Find the solution to the differential equation above.

   (b) How many bacteria are there after 3 hours?

   (c) What is the growth rate of the bacteria after 3 hours?

   (d) Write the equation of the tangent line to $P(t)$ for $t = 3$, and use it to estimate the number of bacteria in the culture after 4 hours.

   (e) Is your estimate from the previous part an over-estimate or an under-estimate? Explain your answer by sketching the graph of $P(t)$.