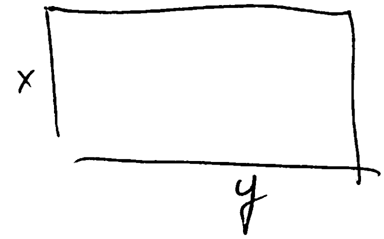
2. (a) objective function $A = xy$



(b) constraint:

$$300 = 2x + 2y$$

(c)

$$300 - 2x = 2y$$
$$150 - x = y$$

$$A(x) = x(150 - x)$$
$$= 150x - x^2$$

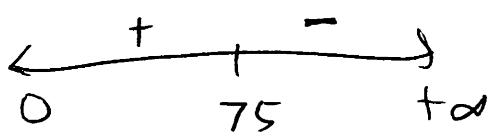
- Intervals of increasing / decreasing

$A'(x) = 150 - 2x$

$0 = 150 - 2x$

$150 = 2x$

$75 = x$



increasing: $(0, 75)$

Decreasing $(75, +\infty)$

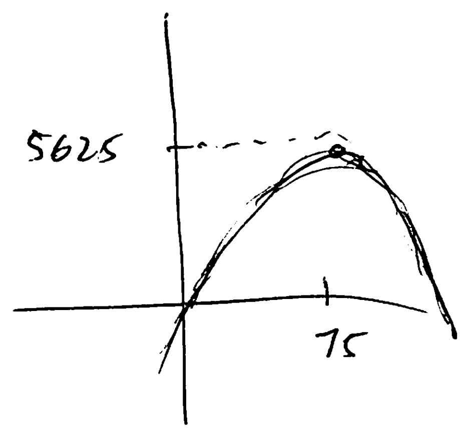
Relative max: $x = 75$

$A(x) = 5625$

Concavity: $A''(x) = -2$

concave down $(0, +\infty)$

(d)



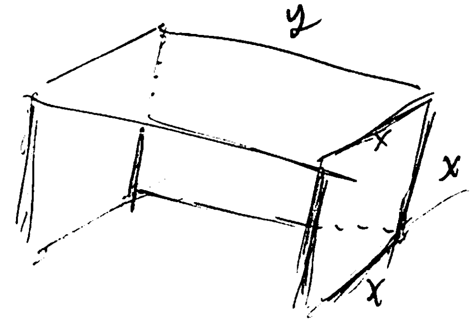
(e) Abs max: $x = 75$

$y = 75$

area $= 5625$

3.

(a) obj function = surface area



$$SA = \underbrace{x \cdot y}_{\text{top}} + \underbrace{x \cdot y}_{\text{back}} + \underbrace{x^2 + x^2}_{\text{2 sides}}$$

$$= 2xy + 2x^2$$

(b) constraint : $x^2 y = 250$

(c) $y = \dfrac{250}{x^2}$

$$SA(x) = \frac{2x \cdot 250}{x^2} + 2x^2$$

$$SA(x) = \frac{500}{x} + 2x^2$$

Increasing / Decreasing:

$$SA'(x) = -\frac{500}{x^2} + 4x$$

$$0 = -\frac{500}{x^2} + 4x$$

$$\frac{500}{x^2} = 4x$$

$$\frac{500}{4} = \frac{4x^3}{4} \longrightarrow 125 = x^3$$

$$\underline{5 = x}$$

$-$ | $+$

0 | 5 | $+\infty$

decreasing: $(0, 5)$

Increasing: $(5, +\infty)$

Rel. max : $x = 5$

$SA(5) = 150$

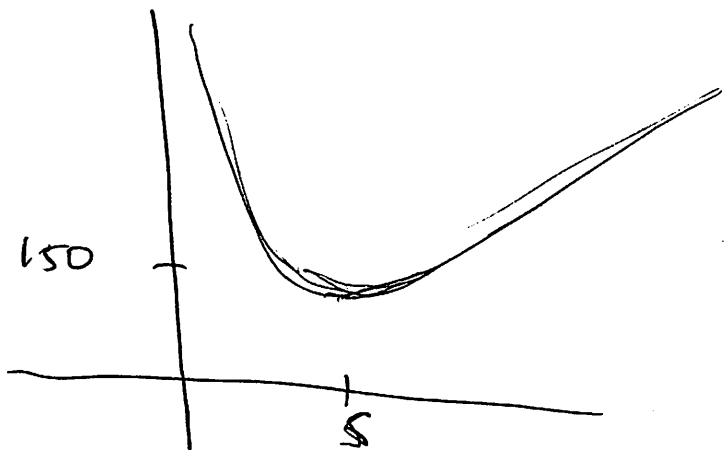
(c)

3. concavity:

$$S''(A) = \frac{1000}{x^3} + 4$$

when $x > 0$, $S''(A)$ always positive.

Therefore $S(A)$ concave up $(0, +\infty)$



Abs min: $x = 5$ $SA = 150$