1.

(a) $f(x) = \left(\frac{\sqrt{2}}{x}\right)^{-9}$

$= \left(2^{1/2} \cdot x^{-1}\right)^{-9}$ distribute -9

$= 2^{-9/2} \cdot x^{9}$

$$\boxed{f'(x) = 2^{-9/2} \cdot 9x^{8}}$$

$$\boxed{f''(x) = 2^{-9/2} \cdot 9 \cdot 8x^{7}}$$

(b) $f(x) = \frac{1}{3x^{3}} + 7x$

$= 1/3 \cdot x^{-3} + 7x$

$$\boxed{f'(x) = 1/3 \cdot (-3) \cdot x^{-4} + 7}$$ → simplified: $\underline{-x^{-4} + 7}$

$$\boxed{f''(x) = 1/3 \cdot (-3) \cdot (-4) \cdot x^{-5}}$$ → simplified: $\underline{4x^{-5}}$

$$f(x) = 3\sqrt[5]{e^2 - 11x}$$

$$f(x) = 3(e^2 - 11x)^{1/5}$$

$$\boxed{f'(x) = 3 \cdot 1/5\,(e^2 - 11x)^{-4/5} \cdot (-11)}$$ ← Don't forget -11 from the general power rule.

$$\boxed{f''(x) = \left(\frac{-33}{5}\right)\left(-4/5\right) \cdot (e^2 - 11x)^{-9/5} \cdot (-11)}$$

(d) $f(x) = (\pi x + 17)^7$

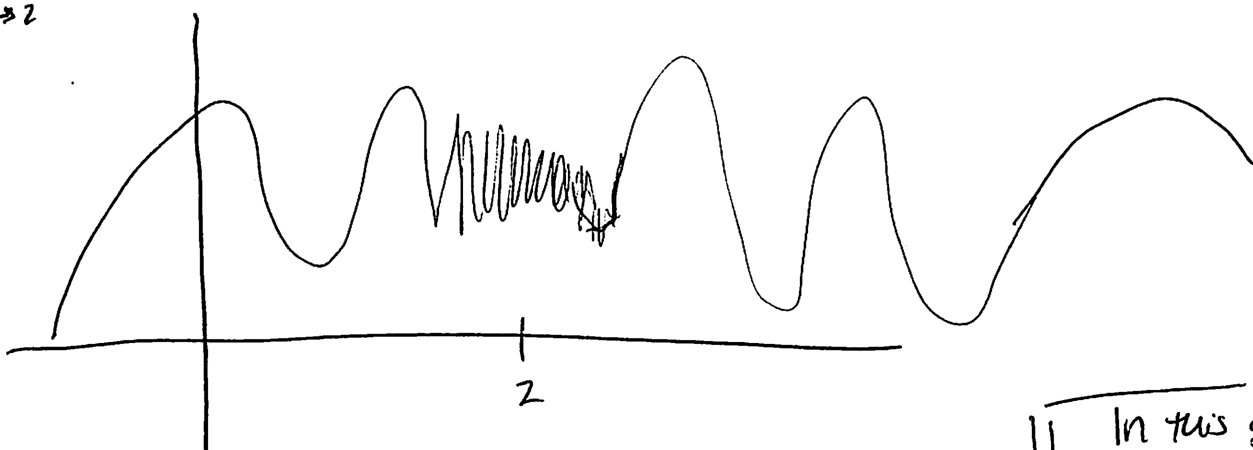
$$\boxed{f'(x) = 7(\pi x + 17)^6 \cdot (\pi)}$$ ← Don't forget π from the general power rule

$$\boxed{f''(x) = 7 \cdot 6 \cdot \pi\,(\pi x + 17)^5 \cdot (\pi)}$$

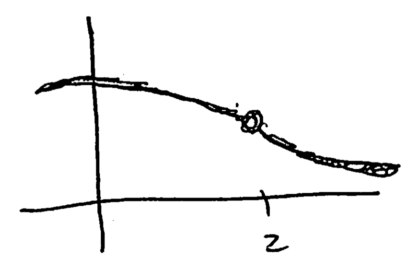
2. The limit of $f(x)$ as $x \to 2$ does not exist:

some examples:

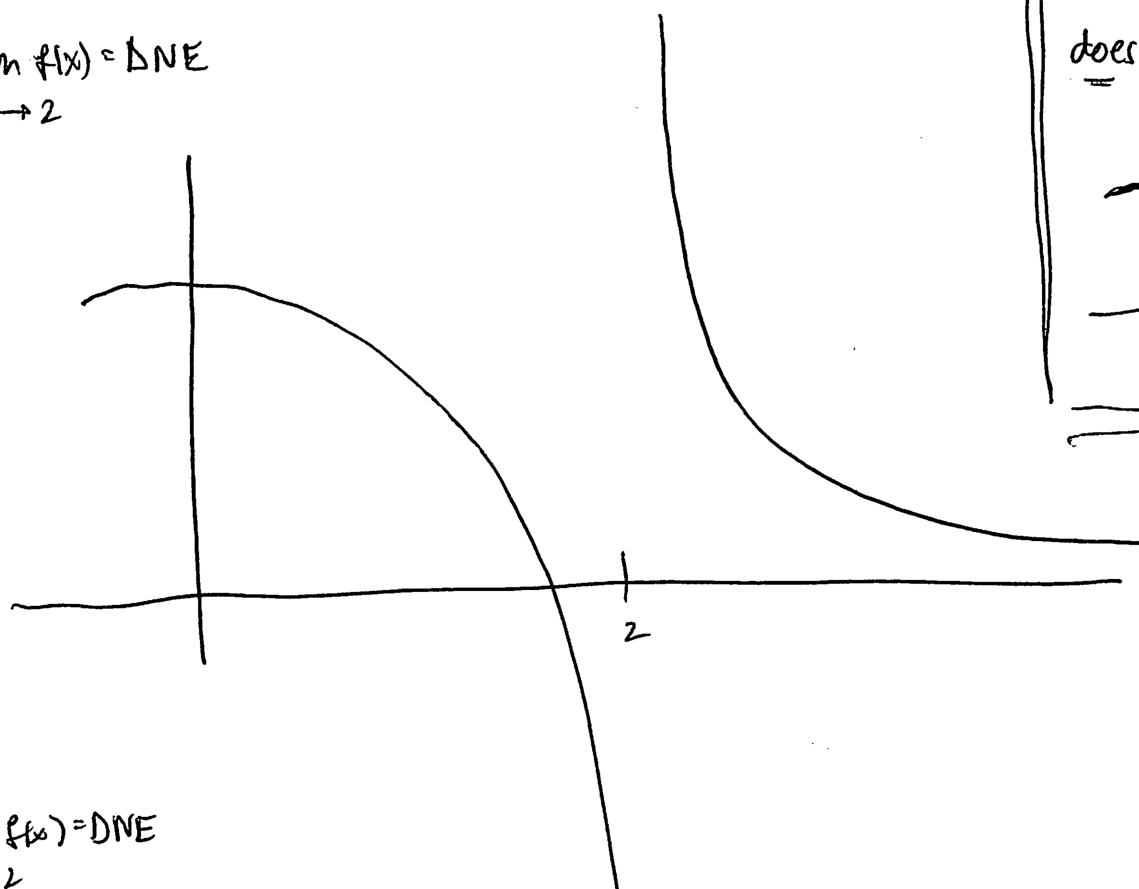
$\lim_{x \to 2} f(x) = DNE$



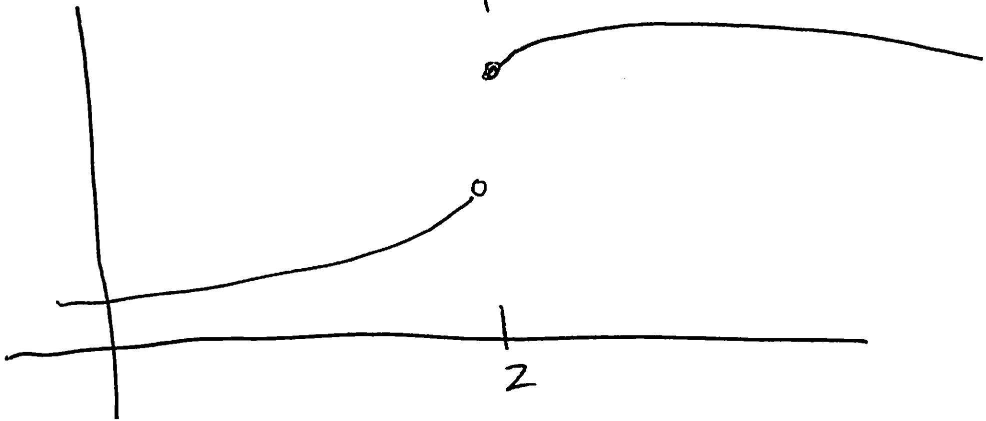
side note:
In this graph, the limit of $f(x)$ as $x \to 2$ does exist.



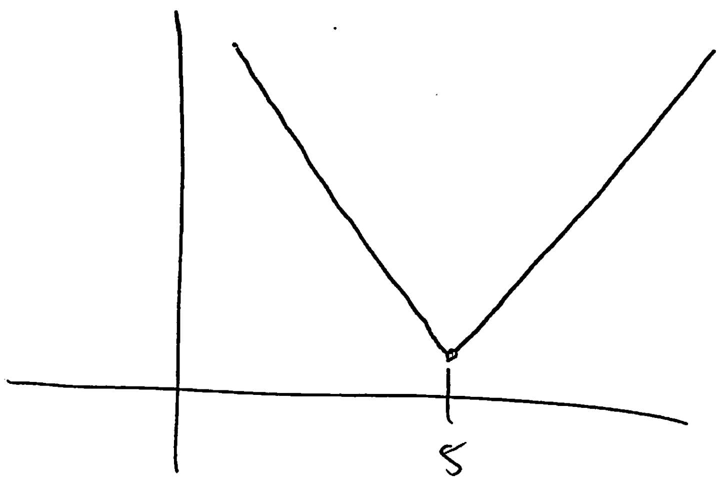
$\lim_{x \to 2} f(x) = DNE$



$\lim_{x \to 2} f(x) = DNE$

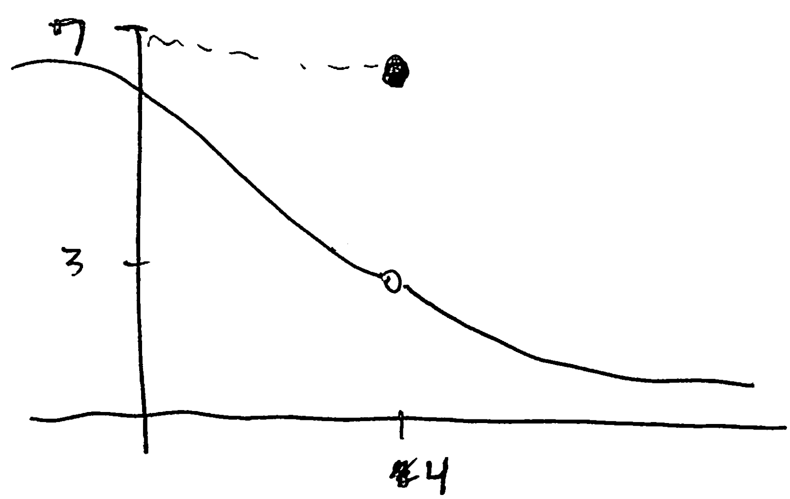


2b f(x) is continuous at $x = 5$, but not differenciable



f(x) has no
no holes are jumps
at $x = 5$; but it does
have a sharp point.

2.(c) f(x) is continuous at $x = 4$ if : $\lim_{x \to 4} f(x)$ exists and is equal to $f(5)$



In this graph $\lim_{x \to 4} f(x)$ exists and is equal to 3.

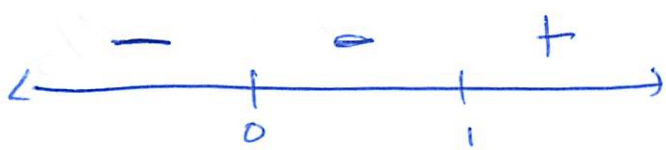
But $f(4) = 7$. Since $3 \neq 7$, f(x) is <u>not</u> continuous.

3.(a) $\quad f(x) = x^4 - (4/3)x^3$

$$f'(x) = 4x^3 - 4/3 \cdot 3x^2$$
$$= 4x^3 - 4x^2$$

$$f'(x) = 0 \longrightarrow \quad 0 = 4x^3 - 4x^2$$
$$0 = 4x^2(x-1)$$

$$\underline{x = 0,\ x = 1}$$



$$f'(x) = 4x^2(x-1)$$

- -1 : $\underset{+}{4(-1)^2}\underset{-}{(-1-1)}$

- $1/2$: $\underset{+}{4(1/2)^2}\underset{-}{(1/2-1)}$

- 2 : $\underset{+}{4(2)^2} \cdot \underset{+}{(2-1)}$

Increasing: $(1, \infty)$

Decreasing: $(-\infty, 1)$

B. Because f' changes from neg to pos. relative minimum at $x = 1$

$f(1) = 1 - 4/3 = -1/3$

min $= (1, -1/3)$

(c) $f''(x) = 4 \cdot 3x^2 - 4 \cdot 2x$

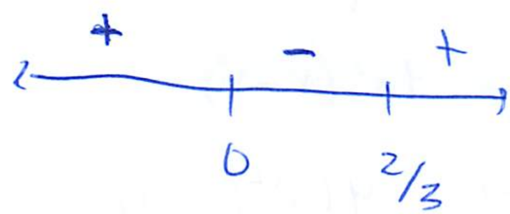
$f''(x) = 12x^2 - 8x$ →

$0 = f''(x)$

$0 = 12x^2 - 8x$

$0 = 4x(3x-2)$

$x = 0, \ x = 2/3$



Test in $f''(x) = 4x(3x-2)$

concave up:

$(-\infty, 0) \cup (2/3, +\infty)$

• -10 : $4(-10)(3(-10)-2)$
neg neg

concave down

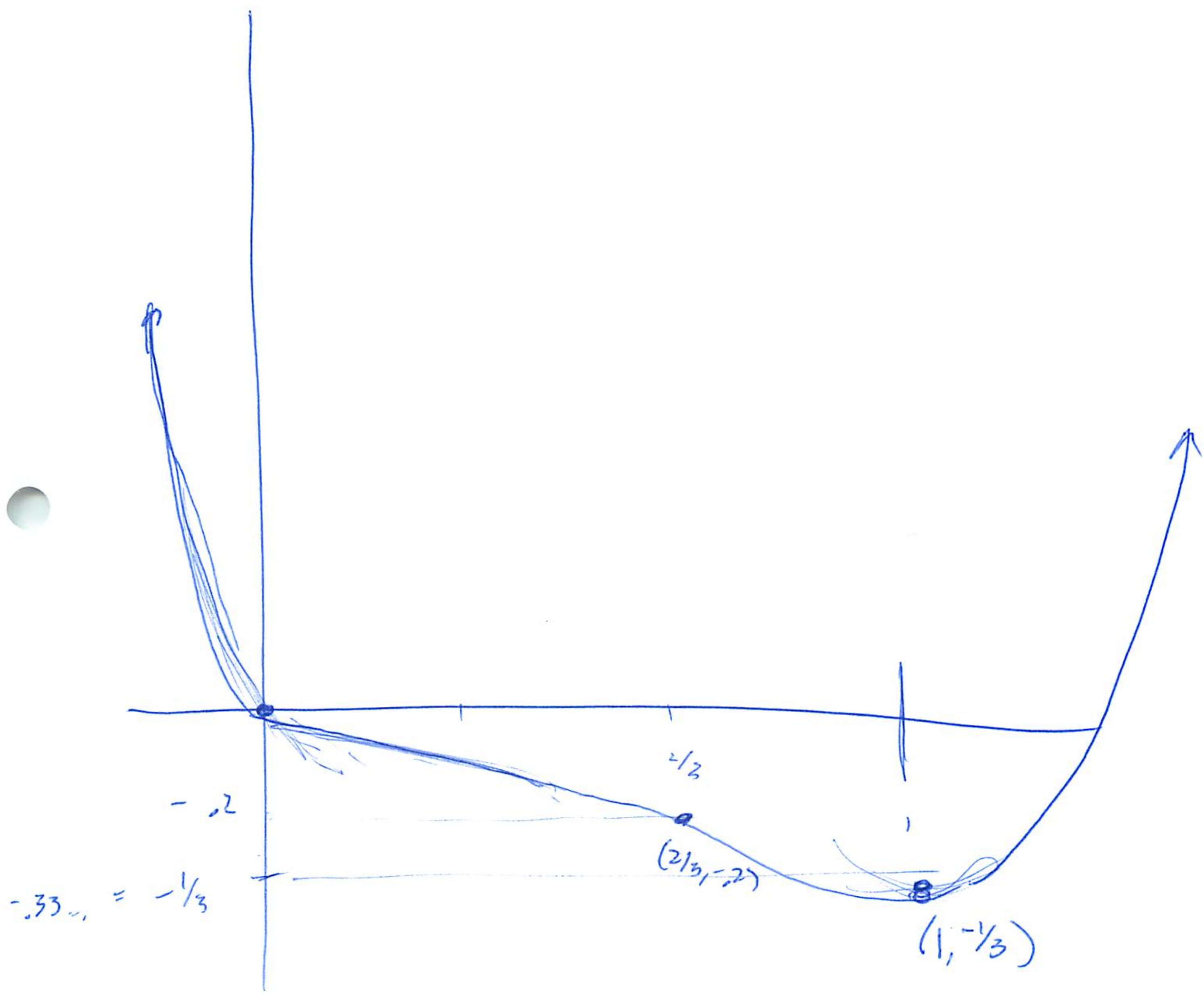
$(0, 2/3)$

• $1/3$: $4(1/3)(3 \cdot 1/3 - 2)$
pos neg.

• 10 : $4(10)(3 \cdot 10 - 2)$
pos pos

3 (d) Inflection pts: • $x = 0$ $f(0) = 0$ $(0,0)$

• $x = 2/3$ $f(2/3) \approx -.1975$



Review
Test solution

#4. a Because $f'(5) < 0$, the amount of drug in the blood-stream is decreasing.

(b) At $t \approx 2.6$ and at $t \approx 7$

(c) Because $f''(5) > 0$, the graph for $f(t)$ is concave up.

(d) • Maximum happens when $f'(t)$ changes from positive to negative.
• Happens at $t = 2$.

Amount of drug in blood stream is maximized at $t = 2$.

(e) A minimum for $f'(t)$ happens when its derivative changes from negative to postive.
Happens at $t = 4$