

Help for # 5 from Derivatives II in Weba assign

Example: Suppose $f(1) = 5$ and $f'(1) = 3$

Approximate the value of $f(1.001)$.

Steps: compute the tangent line at $x=1$.

$$y - y_0 = m(x - x_0)$$

$$x_0 = 1$$

$$y_0 = f(1) = 5$$

$$m = \text{derivative at } 1$$

$$= f'(1) = 3$$

Therefore the eqn of the tangent line is:

$$y - 5 = 3(x - 1)$$

Next: plug in 1.001 into the eqn of the tangent line.

$$y - 5 = 3(1.001 - 1)$$

$$y = 3(.001) + 5$$

$$= 5.003$$

Final Ans: 5.003

For the second part of the problem:

$$g(x) = (f(x))^3$$

$$g(1) = (f(1))^3 = 5^3 = 125$$

$g'(1) = ?$ Use general power rule:

$$g'(x) = 3 f(x)^2 \cdot f'(x)$$

$$\begin{aligned} g'(1) &= 3 \cdot f(1)^2 \cdot f'(1) \\ &= 3 \cdot 5^2 \cdot 3 \end{aligned}$$

Because $f(1) = 5$ and $f'(1) = 3$

$$\begin{aligned} g'(1) &= 9 \cdot 25 \\ &= 225 \end{aligned}$$

Find eqn of tangent line:

$$y - y_0 = m(x - x_0)$$

$$x_0 = 1$$

$$y_0 = g(1) = 125$$

$$m = g'(1) = 225$$

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$$y - 125 = 225(x - 1)$$

Plug in 1.001 as before.