Difference Equations

Graphs and Word Problems

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Graph Example

For each property, list the properties that accurately describe the vertical direction and long term behavior of the graph.

- Monotonic
- Increasing
- Decreasing
- Constant
- Unbounded
- Oscillating
- $|a| < 1$

Graph Examples

Sketch the graphs for the following difference equations:

1. $y_n = .2y_{n-1} + 4.8$, with $y_0 = 1$
2. $y_n = -.8y_{n-1} + 9$, with $y_0 = 50$
3. $y_n = 1.4y_{n-1} - 8$, with $y_0 > 20$, with $y_0 < 20$

At home exercise: Population

Under ideal conditions a bacteria population satisfies the difference equation $y_{n+1} = 1.4y_n$, $y_0 = 1$, where y_n is the size of the population (in millions) after n hours.

Question

Sketch the graph of the solution to the difference equation.

Example: Long-term loan

Suppose that the yearly interest rate on a mortgage is 9% compounded monthly and that you can afford to make payments of $300 per month.

Question

1. Sketch the graph when $y_0 = 1,000,000$. Will you ever pay off the loan?
2. Sketch the graph when $y_0 = 10,000$. Will you ever pay off the loan?

Example: Long-term loan

Suppose that the yearly interest rate on a mortgage is 9% compounded monthly and that you can afford to make payments of $300 per month.

Question

1. Sketch the graph when $y_0 = 1,000,000$. Will you ever pay off the loan?
2. Sketch the graph when $y_0 = 10,000$. Will you ever pay off the loan?
3. What is the largest amount of money you can borrow, and still pay off the loan eventually? (Round to the nearest dollar.)

Example: Long-term savings

A person makes an initial deposit of y_0 dollars into a savings account paying 6% interest compounded annually. He plans to withdraw \$120 at the end of the year.

Question

What is the smallest value of y_0 so that the money will never run out? Round your answer to the nearest dollar.

Algebraic Example

Suppose that you want to take out a mortgage. You can afford to pay $300 per month and the yearly interest rate is 9% compounded monthly.

Question

Exactly how much can you borrow if the mortgage is to be paid off in 30 years?

Savings example

Suppose that you open a savings account, with an annual interest of 8%, compounded quarterly. You don't have any money to deposit at the time when you open the account.

Question

How much money should you deposit at the end of each quarter so that you have $10,000 after 15 years?