

# Difference Equations

## Explicit solution and graphs

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## Example: Difference Equations

Suppose that you have \$40 in a saving account which earns 6% interest, compounded annually.

### Question

Write a recursive formula for the amount of money in the account after  $n$  years.

## Example: Difference Equations

Suppose that you have \$100 in a saving account which earns 4% interest, compounded annually. At the end of each year a \$3 withdrawal is made.

### Question

Write a recursive formula for the amount of money in the account after  $n$  years.

## Example

Suppose that you take out a loan to buy a car. You borrow \$5000 at a 12% interest rate, compounded monthly, and you have payment schedule of \$200 a month.

### Question

Write a recursive formula for the amount of money you owe after  $n$  months.

## Population Example

Suppose that the population of a certain country is currently 6 million. The growth of this population attributable to an excess of births over deaths is 2% per year. Further, the country is experiencing immigration at the rate of 40,000 people per year.

### Question

Write a difference equation to model the population per year for this country.

(Hint: let  $y_n$  = population in year  $n$ , and set  $y_0 = 6$  million.)

- 1 What is  $a$ ?
- 2 What is  $b$ ?

## Example: Monthly loan payments

Suppose that you take out a loan to buy a car. You borrow \$5000 at a 12% interest rate, compounded monthly, and you have payment schedule of \$200 a month.

### Question

- 1 Write the difference equation for this problem.
- 2 Use your answer to find out how much you will owe at the end of the first year.

## Example: Population

Suppose that the population of a certain country is currently 6 million. The growth of this population attributable to an excess of births over deaths is 2% per year. Further, the country is experiencing immigration at the rate of 40,000 people per year.

### Question

- 1 Write down the solution to the difference equation for this problem.
- 2 Draw a guess at what you think the graph looks like.

# Simple interest

Suppose that \$40 is deposited at 6% simple interest.

## Questions

- 1 How much interest does the account earn?
- 2 Write the difference equation for this example.
- 3 Write the solution to the difference equation.



## Graph Examples

Sketch the graphs for the following difference equations:

①  $y_n = .2y_{n-1} + 4.8$ , with  $y_0 = 1$

②  $y_n = -.8y_{n-1} + 9$ , with  $y_0 = 50$

③  $y_n = 1.4y_{n-1} - 8$ , with  $y_0 > 20$ , with  $y_0 < 20$