1. $y_0 \rightarrow$ solve for this

12% annual, compounded monthly

$$y_n = 1.01 y_{n-1} - 120$$

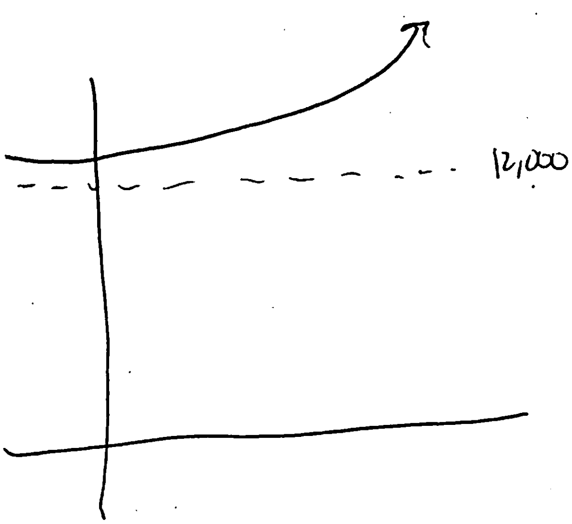
general solution $$y_n = \frac{b}{1-a} + \left(y_0 - \frac{b}{1-a}\right)a^n$$

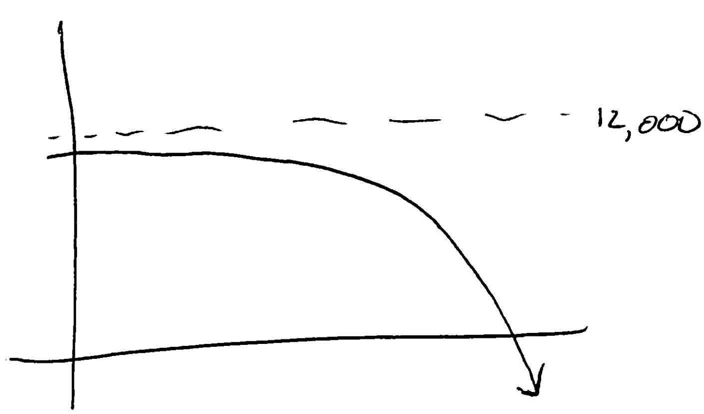
$$y_n = \frac{-120}{1-1.01} + \left(y_0 - \frac{-120}{1-1.01}\right)(1.01)^n$$

$$y_n = \frac{-120}{\frac{-1}{100}} + \left(y_0 - \frac{-120}{\frac{-1}{100}}\right)(1.01)^n$$

$$y_n = 12{,}000 + (y_0 - 12{,}000)(1.01)^n$$

Make 3 graphs. Since $1.01 > 1$ the graph is monotonic. The main thing that will determine the graph is whether $y_0 - \frac{b}{1-a}$ is positive, negative or zero.





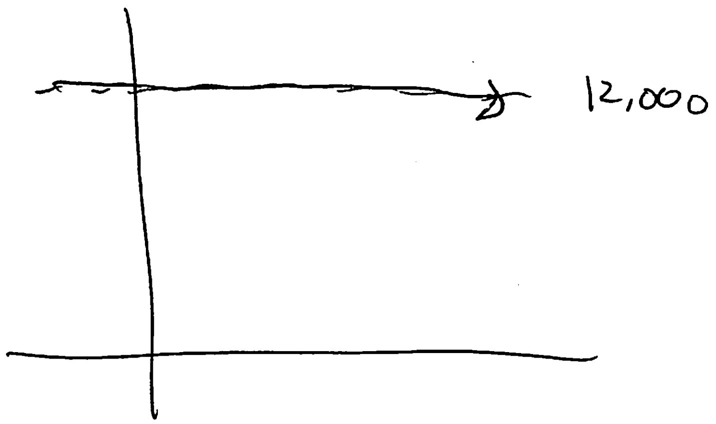
$y_0 - \frac{b}{1-a} > 0$

$y_0 - 12{,}000 > 0$

3rd graph

$y_0 - \frac{b}{1-a} < 0$

$y_0 - 12{,}000 < 0$



$$y_0 - \frac{b}{1-a} = 0$$

$$y_0 - 12{,}000 = 0$$

Which graph has the <u>smallest</u> y_0.

Since we don't want the account to run out of money we can't use $y_0 - 12{,}000 < 0$.

Compare:

$y_0 - 12{,}000 > 0$ vs $y_0 - 12{,}000 = 0$

$\downarrow$

$y_0 > 12{,}000$

$$\underbrace{y_0 = 12{,}000}_{\text{smaller}}$$

$\boxed{\text{Solution deposit } 12{,}000 \text{ initially.}}$

23. $y_0 = 60{,}000$

annual interest 10%, compounded monthly

Difference eqn: $y_n = 1.00833\,y_{n-1} + b$

remember since b is a payment, it should be negative.

Write $a = 1.00833..$ as a fraction

$$a = 1 + \left(\frac{10}{100}\right)\frac{1}{12}$$

$$= 1 + \frac{1}{120}$$

$$= \frac{121}{120}$$

general solution: $y_n = \frac{b}{1-a} + \left(y_0 - \frac{b}{1-a}\right)a^n$

$$y_n = \frac{b}{\frac{120}{120} - \frac{121}{120}} + \left(60{,}000 - \frac{b}{\frac{120}{120} - \frac{121}{120}}\right)\left(\frac{121}{120}\right)^n$$

$$y_n = \frac{b}{-\frac{1}{120}} + \left(60{,}000 - \left(\frac{b}{-\frac{1}{120}}\right)\right)\left(\frac{121}{120}\right)^n$$

$$y_n = -120b + (60{,}000 + 120b)\left(\frac{121}{120}\right)^n$$

plug in $25 \times 12 = 300$ and set eqn equal to zero.

$$0 = -120b + (60{,}000 + 120b)\left(\frac{121}{120}\right)^{300}$$

Don't evaluate until the end

- Distribute $\left(\frac{121}{120}\right)^{300}$

$$0 = -120b + (60{,}000)\cdot\left(\frac{121}{120}\right)^{300} + (120)\left(\frac{121}{120}\right)^{300}\cdot b$$

$$-60{,}000\cdot\left(\frac{121}{120}\right)^{300} = -120b + (120)\cdot\left(\frac{121}{120}\right)^{300}\cdot b$$

factor out b

$$\frac{-60{,}000\cdot\left(\frac{121}{120}\right)^{300}}{\left(-120 + 120\cdot\left(\frac{121}{120}\right)^{300}\right)} = \frac{\left(-120 + 120\cdot\left(\frac{121}{120}\right)^{300}\right)b}{\left(-120 + (120)\cdot\left(\frac{121}{120}\right)^{300}\right)}$$

Divide out the number in front of b

$$b = \frac{(-60{,}000)\cdot\left(\frac{121}{120}\right)^{300}}{\left(-120 + 120\cdot\left(\frac{121}{120}\right)^{300}\right)}$$

$$\approx -545.22$$

Webasign wants the amount of the withdrawal so input 545.22.

Solution to Difference Eqn Word Problems

Q.3 • $y_0 = 500{,}000$

• 8% annual rate, compounded monthly

Difference eqn: $y_n = 1.0066\ldots + b$ Note b is a neg. number

(a) - Write a as a fraction:

$$\left(\frac{8}{100}\right)\frac{1}{12} = \left(\frac{\cancel{4}\cdot 2}{100}\right)\left(\frac{1}{3\cdot\cancel{4}}\right) = \left(\frac{1}{50}\right)\left(\frac{1}{3}\right) = \frac{1}{150}$$

$$a = 1.0066\ldots = 1 + \frac{1}{150} = \frac{151}{150}$$

Write general solution:

$$y_n = \frac{b}{1-a} + \left(y_0 - \frac{b}{1-a}\right)(a^n)$$

leave b as a place holder

$$y_n = \frac{b}{1-\frac{151}{150}} + \left(500{,}000 - \frac{b}{1-\frac{151}{150}}\right)\left(\frac{151}{150}\right)^n$$

Simplify:

$$y_n = \underbrace{\frac{b}{\frac{150}{150}-\frac{151}{150}}}_{} + \left(500{,}000 - \frac{b}{\frac{150}{150}-\frac{151}{150}}\right)\left(\frac{151}{150}\right)^n$$

$$\frac{b}{\frac{-1}{150}} = -b \times (150)$$

$$y_n = -150b + (500{,}000 + 150b)\left(\frac{151}{150}\right)^n$$

4(a) plug in $25 \times 12 = 300$ and set the eqn equal to zero (because after 25 yrs the account is empty)

$$0 = -150b + (500{,}000 + 150b)\left(\frac{151}{150}\right)^{300}$$

Don't evaluate until the very end.

- Now we solve for b.
- Distribute $\left(\frac{151}{150}\right)^{300}$

$$0 = -150b + 500{,}000 \cdot \left(\frac{151}{150}\right)^{300} + (150) \times \left(\frac{151}{150}\right)^{300} \cdot b$$

$$-500{,}000 \cdot \left(\frac{151}{150}\right)^{300} = -150b + (150)\left(\frac{151}{150}\right)^{300} \cdot b$$

Factor out b

$$\frac{-500{,}000\left(\frac{151}{150}\right)^{300}}{\left(-150 + 150 \cdot \left(\frac{151}{150}\right)^{300}\right)} = \frac{\left(-150 + 150 \cdot \left(\frac{151}{150}\right)^{300}\right) \cdot b}{\left(-150 + 150 \cdot \left(\frac{151}{150}\right)^{300}\right)}$$

Divide out # in front of b

$$\frac{(-500{,}000) \cdot \left(\frac{151}{150}\right)^{300}}{\left(-150 + 150 \cdot \left(\frac{151}{150}\right)^{300}\right)} = b$$

When you evaluate this you should get ≈ -3859.08

Note: WebAssign want the amount of the withdrawal. So put positive 3859.08

#6 Recall the general solution is:
3

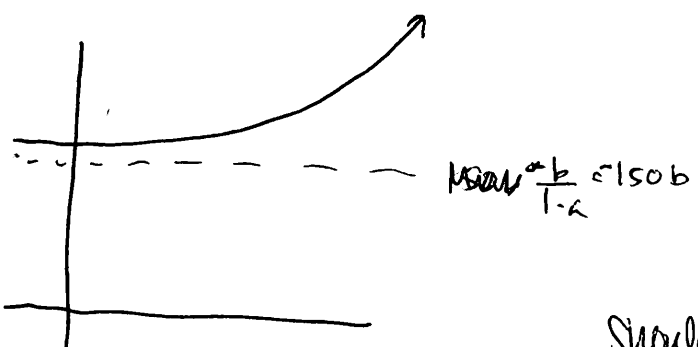
$$y_n = -150b + (500{,}000 + 150b)\left(\frac{151}{150}\right)^n$$

Draw 3 graphs. Remember that b/c $\frac{151}{150} > 1$ the graph is monotonic.

Whether it is increasing, decreasing, constant is completely determined by whether:

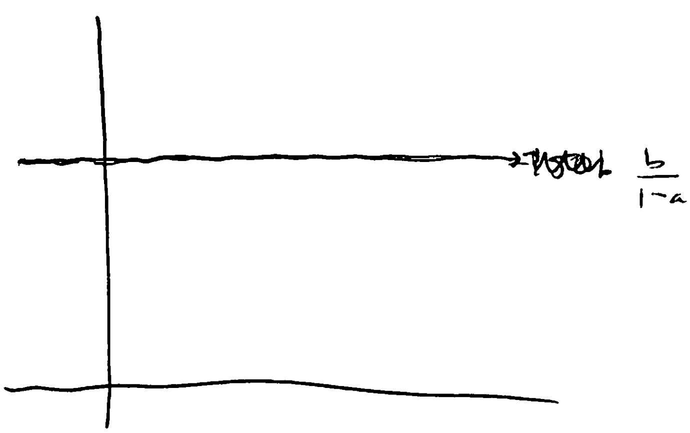
$y_0 - \frac{b}{1-a}$ is positive, neg, or zero.

in this case $y_0 - \frac{b}{1-a} = 500{,}000 + 150b$

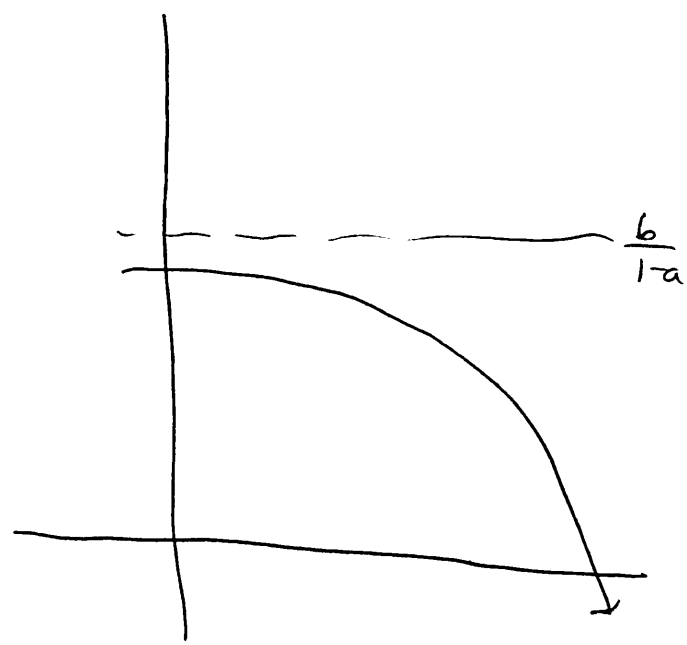


Should correspond to small withdrawal

$,000 + 150b > 0$



$500{,}000 + 150b = 0$



$500{,}000 + 150b < 0$

should correspond w/ a large w/drawal

we don't want our account to run out of money.
So ~~[illegible]~~ the graph w/ $500{,}000 + 150b < 0$ is out.

Determine which has the large withdrawal:

$500{,}000 + 150b > 0$ or $500{,}000 + 150b = 0$

$\downarrow$

$\frac{500{,}000}{-150} > \frac{-150b}{-150}$

$\downarrow$

$\underbrace{-3333.\overline{33} < b}$

Remember that b is negative here.
So, when we say that b (as a negative number) is bigger than -3333.33, that means, ~~[illegible]~~ b is <u>less</u> negative than -3333.33.

$500{,}000 = -150b$

$-3333.33 = b$

This is the bigger withdrawal.